Chapter 2 Measurements

2.4 Prefixes and Equalities

Learning Goal: Use the numerical values of prefixes to write a metric equality.

Prefixes

- A prefix
  - in front of a unit increases or decreases the size of that unit
  - makes units larger or smaller that the initial unit by one or more factors of 10
  - indicates a numerical value

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>kilo</td>
<td>1000</td>
</tr>
<tr>
<td>centi</td>
<td>0.01</td>
</tr>
<tr>
<td>milli</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Metric and SI Prefixes

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Exponential Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>kilo</td>
<td>k</td>
<td>$10^3$</td>
</tr>
<tr>
<td>centi</td>
<td>c</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>milli</td>
<td>m</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>nano</td>
<td>n</td>
<td>$10^{-9}$</td>
</tr>
<tr>
<td>micro</td>
<td>μ</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>millimicro</td>
<td>pm</td>
<td>$10^{-12}$</td>
</tr>
</tbody>
</table>

Metric Equalities

An equality
- states the same measurement in two different units
- can be written using the relationships between two metric units

Example:

1 meter is the same length as 100 cm and 1000 mm.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 m</td>
<td>100 cm</td>
</tr>
<tr>
<td>1 m</td>
<td>1000 mm</td>
</tr>
<tr>
<td>100 cm</td>
<td>1000 mm</td>
</tr>
</tbody>
</table>

Equalities

- use two different units to describe the same measured amount
- are written for relationships between units of the metric system, U.S. units, or between metric and U.S. units

Examples:

<table>
<thead>
<tr>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 m</td>
<td>1000 mm</td>
</tr>
<tr>
<td>1 lb</td>
<td>16 oz</td>
</tr>
<tr>
<td>2.205 lb</td>
<td>1 kg</td>
</tr>
</tbody>
</table>
Exact and Measured Numbers in Equalities

Equalities between units of
• the same system are definitions and are exact numbers
• different systems (metric and U.S.) use measured numbers and count as significant figures

Some Common Equalities

<table>
<thead>
<tr>
<th>Quantity</th>
<th>U.S.</th>
<th>Metric (SI)</th>
<th>Metric-U.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>1 ft = 12 in.</td>
<td>1 km = 1000 m</td>
<td>2.54 cm = 1 in. (exact)</td>
</tr>
<tr>
<td></td>
<td>1 yd = 3 ft</td>
<td>1 m = 1000 mm</td>
<td>1 m = 39.37 in.</td>
</tr>
<tr>
<td></td>
<td>1 mi = 5280 ft</td>
<td>1 cm = 10 mm</td>
<td>1 km = 0.6214 mi</td>
</tr>
<tr>
<td>Volume</td>
<td>1 qt = 4 cups</td>
<td>1 L = 1000 mL</td>
<td>1 L = 1.057 qt</td>
</tr>
<tr>
<td></td>
<td>1 pt = 2 pints</td>
<td>1 L = 100 mL</td>
<td>946.4 mL = 1 qt</td>
</tr>
<tr>
<td></td>
<td>1 gal = 4 qt</td>
<td>1 mL = 1 cm³</td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>1 lb = 16 oz</td>
<td>1 kg = 1000 g</td>
<td>1 kg = 2.205 lb</td>
</tr>
<tr>
<td></td>
<td>1 g = 1000 mg</td>
<td></td>
<td>453.6 g = 1 lb</td>
</tr>
<tr>
<td>Time</td>
<td>1 h = 60 min</td>
<td>1 min = 60 s</td>
<td></td>
</tr>
</tbody>
</table>

Learning Check

Indicate the unit that completes each of the following equalities:
A. 1000 m = _____
   1) 1 mm  2) 1 km  3) 1 dm
B. 0.001 g = _____
   1) 1 mg  2) 1 kg  3) 1 dg
C. 0.01 m = _____
   1) 1 mm  2) 1 cm  3) 1 dm

Learning Check

Complete each of the following equalities:
A. 1 kg = _____
   1) 10 g  2) 100 g  3) 1000 g
B. 1 mm = _____
   1) 0.001 m  2) 0.01 m  3) 0.1 m

Conversion Factors

A conversion factor is
• a fraction obtained from an equality
  Equality: 1 in. = 2.54 cm
• written as a ratio with a numerator and denominator
• is inverted to give two conversion factors for every equality

\[
\frac{1\text{ in.}}{2.54 \text{ cm}} \quad \text{and} \quad \frac{2.54 \text{ cm}}{1\text{ in.}}
\]
Write conversion factors for each pair of units:
A. liters and mL
B. hours and minutes
C. meters and kilometers

Factors with Powers
A conversion factor can be squared or cubed on both sides of the equality.
Equality: 1 in. = 2.54 cm
Equality squared: (1 in.)^2 = (2.54 cm)^2
Equality cubed: (1 in.)^3 = (2.54 cm)^3

Conversion Factors in a Problem
An equality and conversion factors
• may be obtained from information in a word problem
• are for that problem only
Example: The price of one pound (1 lb) of red peppers is $2.39.
Equality: 1 lb peppers = $2.49
Conversion factors: 1 lb red peppers and $2.39 1 lb red peppers

Percent as a Conversion Factor
A percent factor
• gives the ratio of the part to the whole.
  Percent(%) = \frac{\text{part}}{\text{whole}} \times 100
• uses matching units to express the percent
• uses the value 100 and a unit for the whole
Example: A food contains 30% (by mass) fat.
Equality: 100 g food = 30 g fat
Conversion factors: 30 g fat and 100 g food 30 g fat

Percent Factor in a Problem
The thickness of the skin fold at the waist indicates 11% body fat. What equality and percent conversion factors can be written for body fat in kg?
Equality (kg): 100 kg of body mass = 11 kg of fat
Percent conversion factors (kg): 11 kg fat and 100 kg mass

ppm and ppb
Relationships of small percent values are
• ppm (parts per million or mg/kg)
• ppb (parts per billion or \( \mu \text{g/kg} \))
Example: A soil sample contains 2 ppm of lead.
Equality: 2 mg of lead = 1 kg of soil
Conversion factors: 2 mg lead and 1 kg soil
1 kg soil 2 mg lead
Learning Check

Write the equality and conversion factors for each of the following:

A. square meters and square centimeters
B. jewelry that contains 18% gold
C. one gallon of gas costing $3.29
D. a water sample with 55 ppb of chromium (Cr)

Given and Needed Units

To solve a problem:
- identify the given unit
- identify the needed unit

Problem:
A person has a height of 2.0 m. What is that height in inches?

Given unit = meters (m)
The needed unit is the unit for the answer.
needed unit = inches (in.)

Learning Check

An injured person loses 0.30 pints of blood. How many milliliters of blood would that be?

Identify the given and needed units in this problem.
- Given unit = _______
- Needed unit = _______

Guide to Problem Solving Using Conversion Factors

STEP 1 State the given and needed quantities.
STEP 2 Write a plan to convert the given unit to the needed unit.
STEP 3 State the equalities and conversion factors needed to cancel units.
STEP 4 Set up problem to cancel units and calculate answer.

Guide to Problem Solving

The steps in the Guides to Problem Solving Using Conversion Factors are useful in setting up a problem with conversion factors.
Setting up a Problem

How many minutes are 2.5 hours?

**STEP 1**
- **Given**: 2.5 h
- **Need**: min

**STEP 2**
- **Plan**: hours → minutes

**STEP 3**
- **Equalities**: 1 h = 60 min

**STEP 4**
- **Set up problem**
  
  \[
  2.5\, \text{h} \times \frac{60\, \text{min}}{1\, \text{h}} = 150\, \text{min}
  \]

(2 SFs)

Given conversion needed

unit factor unit

Learning Check

A rattlesnake is 2.44 m long. How long is the snake in centimeters?

1) 2440 cm
2) 244 cm
3) 24.4 cm

Using Two or More Factors

- Often, two or more conversion factors are required to obtain the unit needed for the answer.

- Additional conversion factors are placed in the setup to cancel each preceding unit.

- Example:
  
  What is wrong with the following setup?

  \[
  1.4\, \text{day} \times \frac{1\, \text{day}}{24\, \text{h}} \times \frac{1\, \text{h}}{60\, \text{min}}
  \]

  Units = day^2/min, which is not the unit needed. Units do not cancel properly; the setup is wrong.

Using the GPS

What is 165 lb in kilograms?

**STEP 1**
- **Given**: 165 lb
- **Need**: kg

**STEP 2**
- **Plan**: lb → kg

**STEP 3**
- **Equalities/conversion factors**
  
  \[
  1\, \text{kg} = 2.20\, \text{lb}
  \]

  \[
  1\, \text{kg} = 2.20\, \text{lb}
  \]

  (3 SFs)

**STEP 4**
- **Set up problem**

  \[
  165\, \text{lb} \times \frac{1\, \text{kg}}{2.20\, \text{lb}} = 74.8\, \text{kg}
  \]

2.20 lb
**Learning Check**

A bucket contains 4.65 L water. How many gallons of water is that?

**Learning Check**

If a ski pole is 3.0 feet in length, how long is the ski pole in mm?

**Learning Check**

If your pace on a treadmill is 65 meters per minute, how many minutes will it take for you to walk a distance of 7500 feet?

**Percent Factor in a Problem**

If the thickness of the skin fold at the waist indicates 11% of body fat, how much fat in kg does a person have with a body mass of 86 kg?

**Learning Check**

How many lb of sugar are in 120 g of candy if the candy is 25% (by mass) sugar?